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DEVELOPMENT OF A

DECISION RULE FOR SCHEDULING

EXTRA AIRLINE FLIGHT

SECTIONS

A THESIS

Presented to

The Faculty of the Graduate Division

by

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DEVELOPMENT OF A DECISION RULE FOR SCHEDULING EXTRA AIRLINE FLIGHT SECTIONS

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SUMMARY

The objective of this study was to develop a decision rule for scheduling extra airline flight sections for high load factor flights under conditions of risk. A decision rule that considers the following factors was developed:

1. Number of passengers booked on the review date.

2. Conditional profits which would result from either course of action (to add or not to add a second section).

3. Conditional probabilities of having a specified number of passengers on departure.

Expected profit was selected as the criterion for choosing a course of action. Before expressions for the expected profit from each course of action could be written, it was necessary to develop a mathematical model which represents the reservation process and a classification of revenues and costs which influence the expected profit.

It was assumed that sufficient aircraft and flight personnel would be available when needed and could be used elsewhere if a second section were cancelled at departure; additions to the number of reservations booked could be described by the Poisson distribution; cancellations of the reservations could be described by the binomial distribution; a steady state condition existed for the probability of change in the number of reservations and the parameters within the values of interest; sufficient space being available, a passenger will not change

accommodation status at departure; and the number of first-class passengers and the number of tourist-class passengers on departure are independent random variables.

It was concluded from the study that (1) the conditional probability of the number of passengers on departure must be considered in addition to the economic factors involved, (2) for any number of passengers on the review date two courses of action are available, and (3) the model developed can provide an accurate guide for making the decision when the required information is available and the conditions of the model are satisfied.

It was recommended that further study be conducted on the subject in the areas of:

1. Determining a more accurate estimate of passenger revenues.

2. Developing a procedure to provide current estimates of the parameters of the model as they change and a table or other aid to reduce the computations involved in the solution of problems using the proposed decision rule.

3. Measuring the magnitude of error involved in the assumption that the number of passengers on departure are independent random variables when a decision is made not to add a second section.

4. Exploring the simultaneous application of this decision rule to more than one flight.

CHAPTER I

INTRODUCTION

The objective of this study is to develop a decision rule for scheduling extra airline flight sections for high load factor flights under conditions of risk. A review of current literature and discussion with personnel associated with these problems in airline operations indicated a need for improving flight scheduling procedures. This study is intended as a contribution toward this improvement.

On a review day (arbitrarily selected) some time in advance of flight departure, the airline must make a decision as to whether or not to schedule a second section. This decision is made by airline management after considerations of the following principal factors:

1. Potential profit.
2. Type of aircraft with flight personnel available, and position of both in relation to the origin and destination of the flight.
3. Ground service facilities and runway lengths at both origin and destination of flight.
4. Weather forecast for departure and arrival.
5. Demands for second sections on different flights.

In this study only the first factor, potential profit, and its effect on the decision will be considered. Potential profit or expected profit is a weighted average of all the conditional profits of the decision, each conditional profit being weighted by its probability of occurrence. Conditional profit is the absolute profit which would result from the decision and is conditional upon a given number of passengers on departure. This study will formulate a mathematical model to determine these conditional probabilities of having a specified number of passengers on departure, given a certain number of reservations on the review day. In addition, expressions for the conditional profit and expected profit will be written.

Of the remaining factors, numbers two and three are generally deterministic in nature. Information on these factors can be secured by reviewing existing aircraft schedules, crew schedules, or other records which would indicate the existence of facilities and current workload.

Excluding hurricanes and other types of severe storms which can be forecast when located, sudden changes in weather conditions cannot be forecast very far in advance. Therefore, unless severe weather conditions are forecast for departure and arrival, it will be assumed in this study that the weather will not influence the decision when it is made on the review day. It may, however, affect the number of passengers boarding on departure. This effect is considered in the model developed later in the study.

The last factor will also be excluded from this study. If demands for second sections on other flights occurred and only one second section could be added, airline management would consider the competitive advantages in addition to the expected profit in its decision.

With the results of this study, it will be possible to evaluate the expected profit for each flight. The competitive advantages must be evaluated independently of the results of this study by airline management.

Currently airline managements rely to a great extent upon experience and intuition to predict the demand for a second section. There are times of the year, such as certain days just preceding a long holiday weekend, when the need for a second section or more becomes apparent because of the number of passengers who book reservations. At such times a flight may be booked to capacity as early as a month before departure. Airline managements usually recognize these peak travel days and take early steps to have available as many aircraft as possible. This situation will, therefore, not be considered in this study.

There are other times during the year when a second section could have been used, resulting in a higher profit for the flight, if the airline had anticipated the occurrence of an increased demand and scheduled a second section. These events do not occur in a pattern, and airline management must predict the demand, basing the prediction on experience and intuition. It is this situation that will be considered in this study.

The purpose of the proposed decision rule is to augment experience and intuition as well as to provide a method for evaluating the expected profit for the consequences of the decision. The practical application of the proposed rule is to take advantage of these random events and have a second section available when needed.

To simplify the procedure, the decision rule could be stated as follows: On the review day, if the number of first-class and tourist-class passengers is S_1 and S_2 respectively, or more, a second section should be added.

The choice of values for S_1 and S_2 will be determined by considering the expected profit which would be realized if a second section were added when S_1 and S_2 passengers were booked on the review day, and the expected profit which would be realized if a second section were not added. For each combination of values for S_1 and S_2, the decision which yields the best expected profit (best defined as the largest positive or smallest negative value) would be chosen. To satisfy the above statement for the decision, one combination of values for S_1 and S_2 must be selected from the set of values associated with the decision to add a second section. This selection is made by choosing that combination which would yield the desired expected profit.

With an expression for expected profit, airline management must still make a decision for the profit it wishes to make over a long period of time. The level of expected profit established by management will be based upon consideration of its current competitive position in addition to other factors. If, for example, the airline were interested in capturing a larger share of a particular market, it might be willing to accept a negative expected profit. Once the advantage of the increased market for the flight is gained, airline management may decide to accept zero expected profit. In another situation management may deem it necessary to select values for S_1 and S_2 which would provide a high expected profit. This policy would ensure profits but would mean that second sections would only be added in very abnormal situations, a situation that might cause the airline to lose customers to other airlines by not having enough seats available.

The decision rule developed is independent of changes in the economic or competitive position of the airline except as indicated above or as would be reflected in changes in the parameters of the model. Once the level of expected profit has been established and values for S_1 and S_2 have been determined, the decision for second sections can become automatic as long as the conditions of the model are satisfied. Whenever the number of passengers for each class of accommodations on the review day is equal to or greater than S_1 and S_2 respectively, a decision will be made to add a second section. If the number of passengers for either class is less than the assigned value, a second section would not be added.

If the average number of passengers flying daily on a particular flight were increasing and the parameters of the model did not change, the decision to add a second section would be made more often than usual. Eventually, if the trend continued, it would become apparent that two sections should be used regularly and the values for S_1 and S_2 recalculated to determine when a third section would be needed.

If the average number of passengers flying were declining and the parameters did not change, the decision to add a second section would be made infrequently. This would indicate to the airline that the need of a second section had decreased and that, except for abnormal conditions, no review would be necessary.

The review day for the decision will be determined by the airline management after considering its scheduling procedures. The review day must be far enough in advance of departure to permit the airline to schedule available aircraft and flight personnel. This day at the same time should be as close as possible to departure to provide a maximum opportunity for stability in the rate of changes in the number of reservations.

The scheduling of extra sections is not a problem peculiar to the airline industry. All forms of transportation which operate some form of reservation system and have sufficient available equipment for extra sections, and in addition, wish to improve passenger service, need a decision rule for adding second sections. However, many motor bus lines, railroads, and some airlines offering shuttle service operate their service without a reservation system and depend entirely upon their ability to provide sufficient transportation equipment to carry passengers as they request transportation. If equipment is not available, when needed, some forms of transportation are able to crowd a limited number of overflow passengers aboard.

The choice of using an airline to study this problem was based upon convenience. In general, airlines operate a centralized control of reservations and have available the data needed to develop the decision rule and later to use it.

The study will include the determination of the following items:

1. A mathematical model which represents the reservation process.

2. A classification of revenues and costs which influence the expected profit.

3. An expression for the expected profit from each decision.

To aid in the formulation of a mathematical model and the expected profit expressions, it was desirable to make the following assumptions:

1. Sufficient aircraft and flight personnel are available when they are needed and could be used for other flights when the second section is cancelled at departure time.

2. Additions to the number of reservations booked can be described by the Poisson distribution with an estimate of the parameter available.

3. Cancellations of the reservations can be described by the binomial distribution with an estimate of the parameter available.

4. A steady-state condition exists for the probability of change in the number of reservations.

5. The parameters in the model do not change with different values for S_1 and S_2 within the values of interest.

6. If sufficient space is available, a passenger will not change accommodation status at departure.

7. The number of first-class passengers, t_1 or t_1', and the number of tourist-class passengers, t_2 or t_2', on departure are independent random variables.

The effects of relaxing these assumptions are discussed as they appear in the development of the study.

CHAPTER II

REVIEW OF THE LITERATURE

Before summarizing the review of the literature pertinent to this study, a brief discussion of the airline industry will be included to show the importance of customer satisfaction as related to the scheduling of aircraft.

Assuming that the functions of an airline can be summarized, Spears (1) has classified them as speed, safety, and efficiency in the transportation of passengers and cargo. The first two functions, speed and safety, were the primary objectives of the airlines during their early history. Substantial progress in these areas and large financial investments in new equipment now require full consideration for maximum efficiency in flight operations with emphasis on safety. Maintaining schedules is recognized by the airlines to be as important as safety and passenger comfort and is a contributing factor to the growth of public acceptance.

The sources of traffic--mail, passengers, express and freight, and excess baggage--for the airlines were taken from those persons and organizations already utilizing railroads, steamship companies, trucking and bus companies, freight forwarders, and other forms of transportation. The airlines took this traffic business by offering services which were superior to those offered by other forms of transportation at a price which the buyer could afford to pay for the services received.

Since World War II, the total number of passengers carried on domestic flights each year has increased at an average rate of approximately seventeen per cent. At present the domestic airlines account for about forty-four per cent of the total passenger revenue for major domestic carriers.

The effects of mail, express and freight, and excess baggage (which together total only twelve per cent of the airline revenue dollar) will not be studied. In this research they were considered to be unimportant factors in the decision to add a second section for passenger service.

The passengers of an airline flight can be grouped into three categories according to their reason for traveling: business, pleasure, and emergency. In a recent survey, Wolfe (2) found that more than ninety per cent of the total passengers belong in the first two categories. Those passengers flying for pleasure have been further subdivided by Wolfe (3) into three categories: vacationers, tourists on educational trips, or passengers attending special events such as political, social, or sporting events. The composition of any flight can be either homogeneous or heterogeneous depending upon the month, the day, the hour, and the destination of the flight. As a result of many studies, however, the airlines have been successful in determining the characteristics of their flights and have used this information to plan the over-all schedule for the flights.

The word "reservation" has been defined as "the allotting or the securing of accommodations at a hotel, on a train or boat, etc., as for a traveler." (4) Grossman (5) has studied the reservation problem in airlines and concluded that most seats are reserved for several reasons:

a. Reserved space is part of the first-class accommodations and service demanded by most airline travelers.

b. Airlines have a definite limitation on the amount of pay load that can be carried over varying distances.

c. Airlines must obtain the greatest possible use of the available seats on each airline.

d. Control of available space [seats] is necessary to prevent sale of more space on one or more portions of a flight than is available on that portion.

Reservation systems are usually classified in either of two general types: centralized control or decentralized control. In its purest form, centralized control requires one control point for all reservations. A selling agent must contact this point before confirming any reservation. Decentralized control allocates a fixed number of spaces to each station based on its normal needs. If the number of spaces assigned to any station is sold and others are needed, additional space can possibly be secured from other stations without contacting the central control point. Airlines tend to use the centralized control system because of their great volume on nonstop flights, and the other forms of transportation use the decentralized control system. In practice, however, neither system is operated in its purest form.

Beckmann (6) studied both systems as applied to airline reservations and found that in terms of a simplified model with a known distribution of passenger demand, a decentralized quota system was as efficient as the prevailing centralized system.

The present centralized control system for airline reservations has evolved over the years from a part time function of a secretary in a sales or traffic office to a full time, three-shift operation. Initially, reservation information was kept in a simple notebook. Today special purpose electronic data processing systems capable of taking care of each passenger's individual flight information are used. These systems handle all the functions and procedures associated with the sale, confirmation, and control of an air travel reservation from the time a potential customer calls for information on a flight to the time the passenger arrives at his final destination. Those airlines without electronic systems have made other improvements in their space handling and reservation procedures. Other forms of transportation have made similar improvements.

The scheduling of airline flights is generally an operation separate from the handling of reservations but relies heavily on the information about changes in passenger loads for each flight. The preparation of master flight schedules is dependent in many ways upon the requirements of the franchise. Aside from these requirements, the airline schedule must be sufficient to provide the expected service while yielding an adequate profit.

Forecasting for passenger demand is still done principally by the extrapolation of historical data adjusted on the basis of judgment. The factors considered in the judgment are the current economic condition of the nation, average annual growth rate for the airline, seasonal effects, and changes in service by competitors. Some researchers, however, have attempted to find other methods for forecasting. Platt (7) in 1946 was one of the first to develop and publish a mathematical relationship of intercity population, intercity distance, intercity communication, and present carrier service to be used for evaluating intercity air traffic. Richmond (8) also attempted to forecast intercity traffic when he tried to correlate various measures of communications between two cities with their intercity air traffic. Later, the Port of New York Authority (9), using market survey techniques for the first time in passenger forecasts, forecast air travel for the period 1965 to 1975.

The most recent works by Thompson (10) and Beckmann and Bobkoski (11) were the only material found to be suitable for this study. These authors attempted to fit frequency distributions to customer demand and cancellations as they developed for an individual flight. The other methods of forecasting listed above were not suitable because they would only provide an estimate of demand and did not provide a means for evaluating the probability of any estimated demand occurring.

Aside from the graphical methods for scheduling, the principal technique used for scheduling has been linear programming. Manne (12), Vaswani (13) and others have studied the problem of scheduling or allocating a fixed number of aircraft to satisfy a known demand. Ferguson and Dantzig (14) have studied a similar problem but under conditions of uncertain customer demand. Other studies have been made to schedule equipment for other forms of transportation, but these studies were concerned with a fixed number of vehicles and a known demand.

The author was unable to find published reports describing methods for scheduling extra sections. Extra sections are occasionally used, but as mentioned previously the decision is made by relating past experience to an existing situation with no means for evaluating the expected profit.

Controlled overbooking, the problem most closely related to the adding of extra sections, has been studied by Thompson (15) and Bechmann (16). Thompson, using the Poisson distribution as an approximation for passenger demand and the binomial distribution for cancellations, developed an expression which allowed him to prescribe the number of passengers to overbook during three intervals of time before departure. In addition, he studied the possible financial losses which might be incurred through using the recommendations for overbooking. Beckmann found that demand for reservations of addition passengers as well as cancellations are well approximated by gamma distributions. With this information, he developed an expression for the unconditional expected loss and provided a scheme for determining the admissible oversales.

CHAPTER III

THE MODEL EQUATIONS

Description of the Process

The airline reservation process could be described by dividing it into two separate phases as shown in the flow chart, Figure 1. The first phase will include all changes made in the number of reservations for a flight from some time in advance of departure up to two to four hours before flight time. During this interval prospective passengers call the reservation agents, inquire about available flights, and secure a reservation if suitable arrangements are available. Once the reservation has been made, the passenger has three alternatives. He may keep his reservation, he may cancel it at any time, or he may allow the reservation to be canceled by the airline by failing to pick it up during a specified time limit. The rule requiring a passenger to pick up his ticket within a given time limit may not be enforced by the airline if it would affect its competitive position. The cancellation of a reservation by either method is influenced by the passenger's desires; therefore, these two methods of cancellation will be treated as one in this study.

The second phase of the reservation process covers the period two to four hours before departure. At the beginning of this period the control of reservations for the flight is transferred from the central control office to departure control at the airport. This is done by sending a passenger list to the departure control. During the last few hours

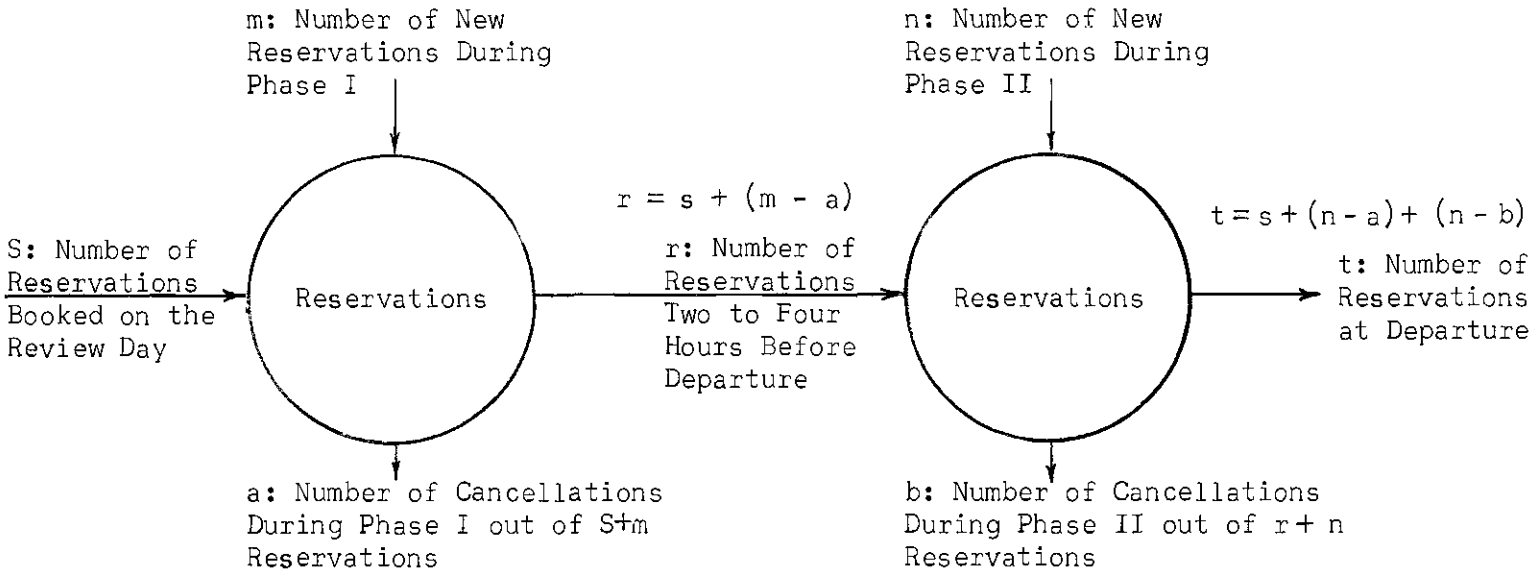

Figure 1. Diagram of the Airline Reservation Process

before departure the number of reservations will change. Losses will be caused by passengers cancelling their reservations at the last moment or failing to appear. Additions will result from correction of errors in the original passenger list and new reservations. All losses during this phase will be treated in this study as one source of cancellations, and all additions will be treated as one source of new reservations.

Although the airline may be unable to accommodate all of the people wanting seats for a flight, it may be able to maintain prospective passengers by means of a waiting list. However, in practice, the frequent use of a waiting list will tend to create a condition whereby prospective passengers will either refuse to be added to the list or will not attempt to make a reservation with the airline. Losses of prospective passengers will also be created when those assigned to a waiting list are unable to secure reservations. Except for abnormal conditions, with two sections available, a waiting list for a flight will not exist.

The purpose of dividing the reservation process into two separate phases is to amplify the effect of the various conditions causing changes in the number of reservations. The probability of either an addition or cancellation should understandably not be the same in both phases, and considerations of this fact should improve the ability of the proposed model to describe the process. Additional subdivisions may be necessary for a particular situation.

The airline reservation process is analogous to several other processes in the physical world, such as arrivals of telephone calls at an exchange and emissions of particles from a radioactive source. The essential feature of this type of process is that events occur from time to time at more or less irregular intervals, the number occurring in any given interval following some probability distribution.

Bechmann as well as Bobkowski and Thompson studied airline reservation bookings and cancellations and agreed that they could be considered random events, but these men differed in their choice of an appropriate probability distribution to describe these events. Bechmann and Bobkowski fitted gamma distributions to demand distributions whereas Thompson found evidence in his study that the demand was markedly negatively skewed and therefore fitted the Poisson distribution. Thompson suggests that his may be caused by the flight having a higher overall load factor.

Since the decision rule from this study is applicable for high load factor flights also, the probability distribution for demand will be assumed to relate to the Poisson distribution. This assumption is further justified by the nature of the reservation process. Since the number of potential customers is large and the probability of anyone desiring to go on a specific flight is small, then the number of reservations already booked does not affect the probability of further requests for a reservation. Feller (17) has shown that if events occur at random and independently in time at a constant rate, the number of events in intervals of the same length follows a Poisson distribution.

In the case of cancellations, Bechmann on the one hand and Bobkowski and Thompson on the other differed in their findings for a suitable theoretical distribution to describe the process. Bechmann and Bobkowski fitted the gamma distribution whereas Thompson found the cancellation rate to be binomial. In this study the binomial distribution will be considered because cancellations for a given flight can only occur from a given number N of confirmed reservations. This situation is equivalent to N Bernoulli trials with a constant probability p of success, a "success" in this case being a cancellation.

One additional assumption for the model of the reservation process is that a steady-state condition exists within each phase of the process. Thompson investigated this assumption with reservation data for intervals of 7, 5, and 2 days and concluded that his process could be considered stationary. A departure from this assumption will be the occasional lack of independence of events. This could occur in the case of block bookings and cancellations, panic cancellations due to political conditions or an air crash, anticipative bookings for a holiday or sports events, and changes in air service. Each of these conditions would be recognizable by the airline and corrective action would be taken to compensate in the decision to add or cancel the second section.

To formulate a model of the airline reservation process encompassing the factors described above, one should consider the general case with only one type of passenger when a decision has been made to add a second section. The model will reflect changes in the number of passengers made in both phases, the formulation beginning with Phase I.

Phase I. The following notation will be used.

λ_1 = average number of new reservations during Phase I

α_1 = probability of a particular passenger's cancelling during Phase I

S = number of reservations at the beginning of Phase I

M = number of new reservations during Phase I

A = number of cancellations during Phase I

R = number of reservations at the end of Phase I

C = capacity of both sections

The probability of m new applications for reservations during this phase is given by the following expression:

$$P(M = m) = \frac{e^{-\lambda_1}(\lambda_1)^m}{m!} \qquad (m = 0,1,2, \ldots, \infty) \tag{1}$$

It is understood that only the first $C - S$ applicants would be given reservations.

The conditional probability of two cancellations out of $m + S$ reservations can be written

$$P(A = a|m + S) = \binom{m + S}{a} \alpha_1^a(1 - \alpha_1)^{m+S-a} \tag{2}$$

$$(a = 0,1,2, \ldots, m + S), \text{ for all } m.$$

The joint probability of m new reservations for a total of $m + S$ reservations and a cancellations during the phase will thus be the product of the probabilities for the two events, i.e.,

$$P(M = m \text{ and } A = a|m + S) \tag{3}$$

$$= P(M = m) \cdot P(A = a|m + S)$$

$$= \frac{e^{-\lambda_1}(\lambda_1)^m}{m!} \binom{m + S}{a} \alpha_1^a (1 - \alpha_1)^{m+S-a}$$

The general joint probability distribution of m and a is represented by Table 1.

If k is the net gain, i.e., $k = m - a$, for this phase the probability of having a total of r reservations where $r = S + k$ is the sum of the probabilities along a diagonal of the general joint probability distribution table where the difference between m and a is equal to k. The conditional probability of R reservations is given by the following general expression:

$$P(R = S + k|S) = \sum_{m=k}^{C-S} P_{m,m-k} \tag{4}$$

$$(\text{for } k = 0, 1, 2, \ldots, C - S)$$

After substitution:

$$P(R = S + k|S) = e^{-\lambda_1}(1 - \alpha_1)^{S+k} \sum_{m=k}^{C-S} \frac{\lambda_1^m}{m!} \binom{m + S}{m - k} \alpha_1^{m-k} \tag{5}$$

<u>Phase II</u>. The following notation will be used in addition to the above:

λ_2 = average number of new reservations during Phase II

α_2 = probability of a particular passenger's cancelling during Phase II

Possible Values of **a**

Possible Values of **m**	0	1	2	. . .	S + 0	S + 1	. . .	C
0	P_{00}	P_{01}	P_{02}	. . .	$P_{0,S+0}$			
1	P_{10}	P_{11}	P_{12}	. . .	$P_{1,S+0}$	$P_{1,S+1}$		
2	P_{20}	P_{21}	P_{22}	. . .	$P_{2,S+0}$	$P_{2,S+1}$		
$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$		
C - S	$P_{C-S,0}$	$P_{C-S,1}$	$P_{C-S,2}$	. . .	$P_{C-S,S+0}$	$P_{C-S,S+1}$	. . .	$P_{C-S,C}$

Table 1. General Joint Probability Distribution of **m** and **a**

N = number of new reservations during Phase II

B = number of cancellations during Phase II

T = number of reservations at the end of Phase II

As in Phase I the probability of n new applications for reservations is given by the following expression:

$$P(N = n) = \frac{e^{-\lambda_2}(\lambda_2)^n}{n!} \qquad (n = 0,1,2, \ldots, \infty) \tag{6}$$

It is understood that only the first $C - k$ applicants would be given reservations.

The conditional probability of b cancellations out of $n + r$ reservations is given by the following expression:

$$P(B = b|n + r) = \binom{n+r}{b} \alpha_2^b(1 - \alpha_2)^{n+r-b} \tag{7}$$

$$(b = 0,1,2, \ldots, C)$$

The joint probability of r reservations at the beginning of Phase II, n new reservations, and b cancellations will thus be the product of the probabilities for the three events. That is

$$P(R = S + k|S \quad \text{and} \quad N = n \quad \text{and} \quad B = b|n + r) \tag{8}$$

$$= P(R = S + k|S) \cdot P(N = n) \cdot P(B = b|n + r)$$

$$= [e^{-\lambda_1}(1 - \alpha_1)^{S+k} \sum_{m=k}^{C-S} \frac{\lambda_1^m}{m!} \binom{m+S}{m} \alpha_1^{m-k}] \times$$

$$\times \left[\frac{e^{-\lambda_2}(\lambda_2)^n}{n!}\right] \cdot \left[\binom{n+r}{b} \alpha_2^b (1-\alpha_2)^{n+r-b}\right]$$

The general joint probability distribution of r, n, and b is represented by Table 2.

If h is the net gain, i.e., $h = k + n - b$, for this phase the probability of having a total of t reservations where $t = S + h$ is the sum of the probabilities along a diagonal of the general joint probability distribution table. The diagonal for any cell in the table is selected where the sum of k and n less b is equal to h. The conditional probability of reservations is given by the following expressions:

$$P(T = S + h|s) = \sum_{k=o}^{h} \sum_{b=o}^{C-S-h} P_{k,h-k+b,b} + \sum_{k=h+1}^{C-S} \sum_{n=o}^{(C-S)-k} P_{k,n,n+k-h}, \qquad (9)$$

(for $h = 0,1,2, \ldots, C-S-1$);

and

$$P(T = S + h|S) = \sum_{k=o}^{h} P_{k,h-k,h-C+S}, \qquad (\text{for } h = C-S) \qquad (10)$$

After substitution for P_{knb}:

$$P(T = S + h|S) = \sum_{k=o}^{h} \sum_{b=o}^{C-S-h} \left\{\left[e^{-\lambda_1}(1-\alpha_1)^{S+k} \sum_{m=k}^{C-S} \frac{\lambda_1^m}{m!}\binom{m+s}{m-k}\alpha_1^{m-k}\right] \right. \qquad (11)$$

$$\left. \cdot \left[\frac{e^{-\lambda_2}\lambda_2^{h-k+b}}{(h-k+b)!}\right]\left[\binom{S+h+b}{b}\alpha_2^b(1-\alpha_2)^{S+h}\right]\right\} +$$

Table 2. General Joint Probability Distribution of r, n, and b

Possible Values of r	Possible Values of n	Possible Values of b: 0	1	2	. . .
	0	$P_{0,0,0}$	$P_{0,0,1}$	$P_{0,0,2}$	. . .
	1	$P_{0,1,0}$	$P_{0,1,1}$	$P_{0,1,2}$	. . .
S+O	.	.	.	.	
	.	.	.	.	
	.	.	.	.	
	C-(S+O)	$P_{0,[C-(S+0)],0}$	$P_{0,[C-(S+0)],1}$	$P_{0,[C-(S+0)],2}$	. . .
	0	$P_{1,0,0}$	$P_{1,0,1}$	$P_{1,0,2}$	. . .
	1	$P_{1,1,0}$	$P_{1,1,1}$	$P_{1,1,2}$	. . .
S+1	.	.	.	.	
	.	.	.	.	
	.	.	.	.	
	C-(S+1)	$P_{1,[C-(S+1)],0}$	$P_{1,[C-(S+1)],1}$	$P_{1,[C^2(S+1],2}$	. . .
.					
.					
.					
S+(C-S-1)	0	$P_{(C-S-1),0,0}$	$P_{(C-S-1),0,1}$	$P_{(C-S-1),0,2}$	. . .
	C-[S+(C-S-1)]	$P_{(C-S-1),\{C-[S+(C-S-1)]\},0}$	$P_{(C-S-1),\{C-[S+(C-S-1)]\},1}$	$P_{(C-S-1),\{C-[S+(C-S-1)]\},2}$	. . .
S+(C-S)	C-[S+(C-S)]	$P_{(C-S),\{C-[S+(C-S)],0}$	$P_{(C-S),\{C-[S+(C-S)]\},1}$	$P_{(C-S),\{C-[S+(C-S)]\},2}$	. . .

(Continued next page)

Table 2 - Continued

Possible Values of r	Possible Values of n	Possible Values of b: S	S + 1	S + 2	. . .
S+0	0	$P_{0,0,S}$			
	1	$P_{0,1,S}$	$P_{0,1,(S+1)}$		
	.	.	.		
	.	.	.		
	.	.	.		
	C-(S+0)	$P_{0,[C-(S+0)],S}$	$P_{0,[C-(S+0)],(S+1)}$	$P_{0,[C-(S+0)],(S+2)}$	. . .
S+1	0	$P_{1,0,S}$	$P_{1,0,(S+1)}$		
	1	$P_{1,1,S}$	$P_{1,1,(S+1)}$	$P_{1,1,(S+2)}$	
	.	.	.	.	
	.	.	.	.	
	.	.	.	.	
	C-(S+1)	$P_{1,[C-(S+1)],S}$	$P_{1,[C-(S+1)],(S+1)}$	$P_{1,[C-(S+1)],(S+2)}$	. . .
.					
.					
.					
S+(C-S-1)	0	$P_{(C-S-1),0,S}$	$P_{(C-S-1),0,(S+1)}$	$P_{(C-S-1),0,(S+2)}$	. . .
	C-[S+(C-S-1)]	$P_{(C-S-1),\{C-[S+(C-S-1)]\},S}$	$P_{(C-S-1),\{C-[S+(C-S-1)]\}(S+1)}$	$P_{(C-S-1),\{C-[S+(C-S-1)]\},(S+2)}$	. . .
S+(C-S)	C-[S+(C-S)]	$P_{(C-S),\{C-[S+(C-S)]\},S}$	$P_{(C-S),\{C-[S+(C-S)]\},(S+1)}$	$P_{(C-S),\{C-[S+(C-S)]\},(S+2)}$	. . .

(Continued next page)

Table 2 - Continued

Possible Values of r	Possible Values of n	Possible Values of b: C - 1	C
	0		
	1		
S+0	.		
	.		
	.		
	C-(S+0)	$P_{0,[C-(S+0)],(C-1)}$	$P_{0,[C-(S+0)],C}$
	0		
	1		
S+1	.		
	.		
	.		
	C-(S+1)	$P_{1,[C-(S+1)],(C-1)}$	$P_{1,[C-(S+1)],C}$
.			
.			
.			
S+(C-S-1)	0	$P_{(C-S-1),0,(C-1)}$	
	C-[S+(C-S-1)]	$P_{(C-S-1),\{C-[S+(C-S-1)]\},(C-1)}$	$P_{(C-S-1),\{C-[S+(C-S-1)]\},C}$
S+(C - S)	C-[S+(C-S)]	$P_{(C-S),\{C-[S+(C-S)]\},(C-1)}$	$P_{(C-S),\{C-[S+(C-S)]\},C}$

$$+ \sum_{k=h+1}^{C-S} \sum_{n=o}^{C-S-k} \left\{ \left[e^{-\lambda_1} (1 - \alpha_1)^{S+k} \sum_{m=k}^{C-S} \frac{\lambda^m}{m!} \binom{m + S}{m - k} \alpha_1^{m-k} \right] \right.$$

$$\left. \cdot \left[\frac{e^{-\lambda_2} \lambda_2^n}{n!} \right] \left[\binom{S + k + h}{n + k - h} \alpha_2^{n+k-h} (1 - \alpha_2)^{S+h} \right] \right\} ,$$

(for h = 0,1,2, ..., C-S-1);

and

$$P(T = S + h|S) = \sum_{k=o}^{h} \left[e^{-\lambda_1} (1 - \alpha_1)^{S+k} \sum_{m=k}^{C-S} \frac{\lambda_1^m}{m!} \binom{m + S}{m - k} \alpha_1^{m-k} \right] \tag{12}$$

$$\left[\frac{e^{-\lambda_2} \lambda_2^{h-k}}{(h - k)!} \right] \left[\binom{k - S}{h - C - S} \alpha_2^{h-C+S} (1 - \alpha_2)^{C} \right] ,$$

(for h = C - S)

It is now possible to formulate a similar expression for the other class of passenger accommodations. The formulation is accomplished by adding a subscript to each of the terms in Equations 11 and 12 to indicate class. Let t_1 be the number of first-class passengers and t_2 be the number of tourist-class passengers.

Because of interest in the probability of having a total of both types of passengers on departure, it is necessary that the joint probability of having t_1 and t_2 passengers on departure be computed.

The event that t_1 will take on any value is independent of the event that t_2 will be any value as long as total available capacity for either class of passenger exceeds the greatest demand which could be normally expected. If this condition were not true, e.g., when all of one class of seats for a flight were reserved, there would be a tendency for prospective passengers to take the other class of accommodation or else secure a seat on another airline if one is available. Violations of this condition would only be true under conditions of abnormal demand; then the airline would make a decision independent of this decision rule. If the increased demand continued over a long period, the airline could then consider using two sections regularly and could make decisions for adding a third section.

The joint probability of having t_1 and t_2 passengers on departure is given by the following general expression, assuming independence:

$$P(t_1 = S_1 + h_1 | S_1 \quad \text{and} \quad t_2 = S_2 + h_2 | S_2) \tag{13}$$

$$= P(t_1 = S_1 + h_1 | S_1) \cdot P(t_2 = S_2 + h_2 | S_2)$$

The general joint probability distribution of t_1 and t_2 is represented by Table 3.

The individual joint probability terms for a particular combination of t_1 and t_2 will be used later to evaluate the expected profit resulting from the decision to add a second section when S_1 and S_2 reservations have been booked on the review day.

Equation 13 was formulated to describe the situation when a decision is made to add a second section with the following assumptions:

1. Additions to the number of reservations booked can be described by the Poisson distribution with an estimate of the parameter available.

Possible Values of t_2

Possible Values of t_1	$S_2 + 0$	$S_2 + 1$	$S_2 + 2$	$\cdots$	$S_2 + (C_2 - S_2)$
$S_1 + 0$	P_{00}	P_{01}	P_{02}	$\cdots$	$P_{0(C_2 - S_2)}$
$S_1 + 1$	P_{10}	P_{11}	P_{12}	$\cdots$	$P_{1(C_2 - S_2)}$
$S_1 + 2$	P_{20}	P_{21}	P_{22}	$\cdots$	$P_{2(C_2 - S_2)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$
$S_1 + (C_1 - S_1)$	$P_{(C_1 - S_1)0}$	$P_{(C_1 - S_1)1}$	$P_{(C_1 - S_1)2}$	$\cdots$	$P_{(C_1 - S_1)(C_2 - S_2)}$

Table 3. General Joint Probability Distribution of t_1 and t_2.

2. Cancellations of the reservations can be described by the binomial distribution with an estimate of the parameter available.

3. A steady-state condition exists for the probability of change in the number of reservations.

4. The number of first-class passengers, t_1, and the number of tourist-class passengers, t_2, on departure are independent random variables.

These assumptions will be used to formulate an expression for the conditional probability of having t_1' and t_2', first-class and tourist-class passengers respectively, by departure given S_1 and S_2, first-class and tourist-class passengers respectively, booked on the review day when a decision is made not to add a second section.

In this situation t_1' and t_2' should truncate at their respective capacity of seats in the aircraft. If this limitation were not imposed, they could take on all possible values representing the potential number of passengers for each class of accommodations for the flight. This limitation will not be applied in this study in order that the lost profit (and other intangible costs created when a prospective passenger is refused a reservation) can be evaluated.

The assumption that t_1' and t_2' are independent random variables is applicable in this situation until one class of accommodations is booked to capacity. The next prospective passenger for that class of accommodations may accept an alternative class rather than having his name added to a waiting list or searching for space on another flight or with another airline. The number of passengers that can accept an alternative class of accommodations is, of course, limited by the remaining space in the other class.

Because sufficient data are not available to properly treat this problem and because it was deemed to be beyond the scope of this thesis, the assumption of independence will be used. Normally, the review day for making the decision for a second section will be before either class of accommodations is booked to capacity and before the second section is added. The error in making this assumption may not be important. Further study with actual data will provide additional information on the validity of this assumption.

With the above assumptions, it is now possible to formulate an expression to describe the joint probability of having a potential combination of t_1' and t_2' passengers on departure even when t_1' is greater than C_1 and t_2' is greater than C_2.

CHAPTER IV

AIRLINE REVENUES AND EXPENSES

The four sources of revenue for the airline include the revenue from transporting passengers, mail, express and freight, and excess baggage. However, passenger revenue will be the only source of revenue considered in the decision to add a second section. The other three sources together only contribute approximately twelve per cent of the airline revenue dollar and were not considered in this study to be important factors in the decision.

The revenue from a passenger can vary according to the type of flight, age of the passenger, choice of seating accommodations, and other special fare programs. These factors which influence the revenue, however, are not available every day for every flight. It is, therefore, necessary to specify the type of flight, day of the week for the flight and possibly the time of departure, and length of stay at the destination in addition to the above factors in order to determine the revenue for a passenger occupying one seat on the aircraft.

Because of the wide variation possible in the revenue, it is customary for the airlines to determine the revenue for a first-class jet flight, such as the type flight considered in this study, by using a weighted-average figure. The figure is determined by considering the first-class and tourist fare and the ratio of the two classes of seating accommodations available in the aircraft. If the flight typically has a large number of children or other passengers taking advantage of available plans which allow the passengers to fly at a reduced rate, the approximate percentage of this type of passenger for each class is also considered. Since accurate information is not currently available to indicate what these percentages should be, approximations would be used.

The operating expenses associated with an airline flight are classified by the Civil Aeronautics Board into two broad categories: Aircraft Operating Expenses and Ground and Indirect Expenses. Aircraft operating expenses include all costs which are directly associated with out-of-pocket expenses of aircraft operation. Ground and indirect expenses include all expenses necessary to provide the ground support for the flight. Table 4 lists the various operating expenses in their proper classifications.

There are relatively few fixed costs involved in the operation of an airline. The majority of the costs could be classified as a constant cost because they cease if the airline suspends operation but do not vary in proportion to changes in the volume of business handled.

For this reason, all costs common to the normal operation of the airline flight will be assigned to the first section for any flight. The costs for a second section will include a constant cost for flight personnel salaries and flight equipment maintenance; a direct variable cost resulting from selling additional tickets, serving of more meals, use of additional fuel, and additional passenger liability insurance; and a cost for ferrying the aircraft and flight personnel, if applicable.

Table 4. Classification of Airline Expenses

TYPE OF EXPENSE

AIRCRAFT OPERATING EXPENSES

Flying Operations
- Captains and Senior Pilots
- First Officers and Copilots
- Aircraft Engine Fuels
- Aircraft Engine Fuel Taxes
- Aircraft Engine Oils (Including Tax)
- Flight Equipment Insurance and Injuries, Loss and Damages
- Flying Liability and Compensation Insurance
- Other Expenses

Flight Equipment Maintenance - Direct
- Aircraft Repairs
- Aircraft Engine Repairs
- Other Expenses

Depreciation - Flight Equipment
- Aircraft Depreciation
- Aircraft Engine Depreciation
- Other Flight Equipment Depreciation

GROUND AND INDIRECT EXPENSES

Ground Operations
- Salaries of Superintendents, Airport and Hanger Employees, etc.
- Rents of Fields, Buildings and Offices
- Other Expenses

Ground Equipment Maintenance - Direct (Total)

Equipment Maintenance - Indirect (Total)

Depreciation - Ground Equipment (Total)

Passenger Service
- Stewards and Stewardesses
- Passenger Supplies and Food Expense
- Passenger Liability Insurance
- Other Expenses

Traffic and Sales
- Salaries of Superintendents, Traffic Managers, Agents, etc.
- Other Expenses

Advertising and Publicity (Total)

General and Administrative
- Salaries of General Officers
- General Office Employees
- Legal Salaries, Fees and Expenses
- Special Professional and Technical Services
- Regulatory Proceeding Expenses
- Pensions and Welfare
- General Taxes (Excludes Income Taxes)
- Other Expenses

Two other costs to be considered, when evaluating the expected profit resulting from making a decision to add a second section, are related to costs which exist if the second section is not used. This event could occur whenever the number of passengers for both classes of accomodations is less than or equal to the capacity of the first section. When this happens, the second section would not be used and it may stand idle.

The first cost to be considered in this situation is the cost incurred by having the flight personnel assemble at the airport and by performing the maintenance necessary to prepare an aircraft for flight. The flight personnel may be sent home but some form of remuneration will be necessary.

The second cost to be considered is created by the opportunity lost to create a profit. An aircraft standing idle at one airport may possess a capacity to produce a profit, if it were available where a demand existed. Sometimes, it may be possible for the airline to transfer the aircraft in time to the other airport and recoup the profit. At other times the aircraft will stand idle and the profit will be lost. The value for this cost must be based upon the experience and knowledge of the airline applying the results of this study.

Many of the revenues and costs described above would also apply in a situation where only one aircraft is used. There is, however, another important cost which must be considered. When the decision is made not to add a second section, it is possible that prospective passengers will be refused reservations because sufficient space is not available. In addition to the lost revenue, which might have been obtained, there are other intangible costs involved. These intangible costs are related to the resulting loss of goodwill and are not readily measurable. The airline must, however, assign a reasonable value if it desires to obtain the best results from the decision rule.

CHAPTER V

EXPECTED PROFIT

The criterion to be used for determining suitable values for S_1 and S_2, when making a decision to add a second section, is expected profit. To simplify the following expressions only gross profit before taxes will be considered. The level of expected profit which the airline wishes to make will be a management decision influenced by competitive conditions discussed earlier.

It is the purpose of this chapter to formulate general expressions for the expected profit from each decision for each combination of S_1 and S_2. The expressions will relate the costs and revenues described in the previous chapter and the probabilistic model of the reservation process formulated earlier.

A basic rule by which the expected profit from the decision can be evaluated is as follows: No passenger will change status of seating accommodations at departure. This means in practice that once any class of seats on the first section is reserved, the second section will be used even though space is available on the first section for additional passengers in another class of accommodations. Relaxation of this rule is possible, but it would require additional processing of boarding passengers to find who would change their status. It would also require some criterion for deciding who should be asked if volunteers could not be found.

If there were a restriction on the total number of seats available and passengers either took a different class of seat or left the aircraft, Thompson (18) found that first-class passengers were generally not willing to be transferred to any empty tourist-class seat; but a tourist-class passenger would not object to being transferred to any empty first-class seat in an emergency. "This seems reasonable," Thompson continues, "since F [first-class] passengers are willing initially to pay the difference between F and T [tourist-class] fares to obtain a better seat and therefore should not be so willing to forego this advantage." (19) In practice, he found that approximately one-half of the first-class passengers would accept an empty tourist seat and, of course, the consequent fare adjustment, also.

The problem considered here is slightly different from Thompson's situation. Throughout the period between the review day and departure, there will generally be no limitation on available seats for either class of passenger. Therefore, the passenger makes an uninhibited decision for accommodations. In addition, the reason for a change in status is not based on a situation involving whether or not the passenger will be left, but is based on whether or not the airline will use the second section. Attempting to force a change in seating accommodations under this condition is not conducive to good passenger relations and could, over a long period of time, result in poor customer relations and subsequent decline in passenger revenues.

The following expressions for conditional profit and expected profit, when a decision has been made to add a second section, will be formulated incorporating this rule.

Conditional Profit

The conditional profit is a measure of the absolute profit which would result if a second section were added and is conditional upon a given event occurring, e.g., t_1, first-class passengers, and t_2, tourist-class passengers, on departure.

The following notation will be used:

S_1 = number of first-class reservations booked at the beginning of Phase I

S_2 = number of tourist-class reservations booked at the beginning of Phase I

t_{1h_1} = number of first-class passengers on departure

t_{2h_2} = number of tourist-class passengers on departure

P_{h_1,h_2} = joint probability of t_{1h_1} and t_{2h_2} passengers departure for S_1 and S_2

C_1 = capacity of both sections for first-class passengers

C_1' = capacity of first section for first-class passengers

C_2 = capacity of both sections for tourist-class passengers

C_2' = capacity of first section for tourist-class passengers

R_1 = revenue from a first-class fare

R_2 = revenue from a tourist-class fare

C = variable cost per passenger

F = constant cost for flight and first section

F' = additional constant cost for second section

G = cost of having a second section when it is not needed plus lost profit from an idle aircraft.

$$P(t_{1h_1},\ t_{2h_2}) \;=\; \text{conditional profit}$$

The conditional profit is given by the following expressions:

$$P(t_{1h_1},\ t_{2h_2}) = (R_1 - C)t_{1h_1} + (R_2 - C)t_{2h_2} - F - G \qquad (14)$$

$$\left(\text{for}\quad \begin{aligned} 0 &\leq t_{1h_1} \leq C_1' - S_1 \\ 0 &\leq t_{2h_2} \leq C_2' - S_2 \end{aligned}\right)$$

and

$$P(t_{1h_1},\ t_{2h_2}) = (R_1 - C)t_{1h_1} + (R_2 - C)t_{2h_2} - F - F' \qquad (15)$$

$$\left(\text{for}\quad \begin{aligned} C_1' - S_1 + 1 &\leq t_{1h_1} \leq C_1 - S_1 \\ C_2' - S_2 + 1 &\leq t_{2h_2} \leq C_2 - S_2 \end{aligned}\right)$$

Expected Profit

The expected profit of the decision is a weighted average of all of the conditional profits of the decision, each conditional profit being weighted by its probability. The general expression can be written as follows:

$$E[P(t_{1h_1},\ t_{2h_2})] = \sum_{h_1=0}^{C_1'-S_1} \sum_{h_2=0}^{C_2'-S_2} \{P_{h_1,h_2}[P(t_{1h_1},\ t_{2h_2})]\} \qquad (16)$$

$$+ \sum_{h_1=C_1'-S_1+1}^{C_1-S_1} \sum_{h_2=0}^{C_2-S_2} \{P_{h_1,h_2}[P(t_{1h_1},\ t_{2h_2})]\}$$

$$+ \sum_{h_1=0}^{C_1'-S_1} \sum_{h_2=C_2'-S_2+1}^{C_2-S_2} \{P_{h_1,h_2}[P(t_{1h_1},\ t_{2h_2})]\}$$

With the above expression, it is possible to evaluate the expected profit of the decision to add a second section for any combination of S_1 and S_2.

Similar expressions will be formulated for the conditional profit and expected profit when a decision has been made not to add a second section.

Conditional Profit

The conditional profit is a measure of the absolute profit which would result if a second section were not added and is conditional upon a given event occurring, e.g., t'_1, first-class passenger, and t'_2, tourist-class passengers, on departure.

The following notation will be used:

S_1 = number of first-class reservations booked at the beginning of Phase I

S_2 = number of tourist-class reservations booked at the beginning of Phase I

t'_{1h_1} = number of first-class passengers on departure including those refused reservations when t'_{1h_1} is greater than C'_1

t'_{2h_2} = number of tourist-class passengers on departure including those refused reservations when t'_{2h_2} is greater than C'_2

C'_1 = capacity of first section for first-class passengers

C'_2 = capacity of first section for tourist-class passengers

R_1 = revenue from a first-class fare

R_2 = revenue from a tourist-class fare

C = variable cost per passenger

F = constant cost for flight and first section

W = cost of goodwill lost when a prospective passenger is refused a reservation

$P(t'_{1h_1}, t'_{2h_2})$ = conditional profit

To stay within a reasonable range of values for t'_{1h_1} and t'_{2h_2}, they are assigned an upper limit equal to the total capacity of space available if two sections were used, e.g., $t'_{1h_1} \leq C_1 - S_1$ and $t'_{2h_2} \leq C_2 - S_2$.

The conditional profit is given by the following expressions:

$$P(t'_{1h_1}, t'_{2h_2}) = (R_1 - C)t'_{1h_1} + (R_2 - C)t'_{2h_2} - F \tag{17}$$

$$\left(\text{for} \quad 0 \leq t'_{1h_1} \leq C'_1 - S_1 \atop 0 \leq t'_{2h_2} \leq C'_2 - S_2\right)$$

and

$$P(t'_{1h_1}, t'_{2h_2}) = (R_1 - C)(C'_1 - S_1) + (R_2 - C)(C'_2 - S_2) - F \tag{18}$$

$$- W[t'_{1h_1} - (C'_1 - S_1)] - W[t'_{2h_2} - (C'_2 - S_2)]$$

$$\left(\text{for} \quad C'_1 - S_1 + 1 \leq t'_{1h_1} \leq C_1 - S_1 \atop C'_2 - S_2 + 1 \leq t'_{2h_2} \leq C_2 - S_2\right)$$

Expected Profit

The general expression for expected profit can be written as follows:

$$E[P(t'_{1h_1}, t'_{2h_2})] = \sum_{h_1=0}^{C'_1-S_1} \sum_{h_2=0}^{C'_2-S_2} \{P_{h_1,h_2}[P(t'_{1h_1}, t'_{2h_2})]\} \tag{19}$$

$$+ \sum_{h_1=C'_1-S_1+1}^{C'_1-S_1} \sum_{h_2=0}^{C_2-S_2} \{P_{h_1,h_2}[P(t'_{1h_1}, t'_{2h_2})]\}$$

$$+ \sum_{h_1=0}^{C'_1-S_1} \sum_{h_2=C'_2-S_2+1}^{C_2-S_2} \{P_{h_1,h_2}[P(t'_{1h_1}, t'_{2h_2})]\} .$$

CHAPTER VI

SAMPLE APPLICATION

Included in the original plan for this study was an application of the results using actual data for a high load factor flight taken from a national airline. Investigation revealed that the airline did not have available sufficient data to provide a thorough application. A second plan was then proposed to apply the results with estimates of the parameters collected from a review of available records. This plan was abandoned when the computation time for the solution was determined. Using the available computer, this plan was not considered economically feasible.

It was then decided to illustrate the applicability of the results of this study with a sample problem where the numbers involved were scaled down proportionately and only one class of passenger accommodations was available. The following hypothetical flight was created with the cooperation of operating people of an airline.

The problem is as follows: to determine value for S which will provide a zero expected profit or closest positive value, if a decision is made to add a second section when S or more passengers have booked reservations on the review day.

For this flight the airline would operate an aircraft with a seating capacity of 15 passengers, or a total capacity of 30 passengers if two aircraft are used. On the average, 10 new reservations are booked during Phase I and three in Phase II. The probability of a particular reservation being cancelled is 0.20 during Phase I and 0.10 during Phase II. The revenue from a passenger is $50 and the variable cost for a passenger on the flight is $5. The fixed costs for the flight and cost for the first section is $375. If a second section were added and used, the additional fixed cost would be $190. If a section were added and not used, the fixed cost would be $225. Loss of goodwill, when a prospective passenger is refused a reservation is estimated to cost three times the revenue, or $150.

This information is summarized below according to the notation of this thesis:

$C'_1 = 15$	$\alpha_1 = 0.20$	$F = \$375.$
$C_1 = 30$	$\alpha_2 = 0.10$	$F' = \$190.$
$\lambda_1 = 10$	$R = \$50.$	$G = \$225.$
$\lambda_2 = 3$	$C = \$\,5.$	$W = \$150.$

The proper value for S, which satisfies the criterion for expected profit, can be determined with a numerical solution of the expressions for expected profit when either decision is made for different values of S. For each value of S, the decision rule which yields the best expected profit (best defined as the largest positive or smallest negative value) would be chosen. From the set of values associated with decision 1, which is to add a second section, the one value of S would be selected where the minimum expected profit is equal to or greater than the desired profit.

Solution of the expressions for expected profit for a particular value of S can be accomplished with one of two methods. The first method requires that the expression be solved algebraically. This method is tedious and time-consuming even for a computer because many values must be determined for the individual terms from the binomial probability distribution.

To reduce the computational time, a second method is available. This method eliminates the necessity of solving Equations 22 and 23. Instead, values for the various probabilities are taken from available tables and substituted in Equations 3 and 8. The results are presented as two and three dimensional matrices. The summations indicated in Equations 4, 9, and 10 are performed and the results substituted in Equations 16 and 19 to determine the value of expected profit from each decision. The principal disadvantage of this method arises from the need for either a large storage capacity in the computer or facilities for retrieving data from outside storage.

The second method was chosen for the solution of the expressions for expected profit for the problem considered. Values for the Poisson distribution were taken from Molina's Tables and for the binomial distribution from the U. S. Army Ordnance Corps' Tables. An Algol routine was written for the Burroughs 220 Electronic Data Processing System to perform the necessary computations (see Appendix).

If the model conditions exist (steady-state, Poisson distribution, etc.) and the information above is given, it is possible to obtain the information listed in Tables 5, 6, 7, 8, and 9.

Tables 5 and 6 are shown to illustrate the changes in the conditional probabilities for different values of r and t respectively, as S increases from 4 to 9. Values for S, other than those listed, were not considered because their associated expected profit would be outside the area of consideration.

Table 9 shows the expected profit from making either decision 1, which is to add a second section, or decision 2, which is not to add a second section, when S passengers have booked reservations on the review day. Since the level of expected profit established for this problem was zero, the appropriate value for S would be 7. If 7 or more passengers have booked reservations on the review day, decision 1 would be made.

Table 5. Conditional Probability of r Reservations at the End of Phase I Given S Reservations Booked on the Review Day

r	S = 4	S = 5	S = 6	S = 7	S = 8	S = 9
4	0.0037	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0115	0.0053	0.0000	0.0000	0.0000	0.0000
6	0.0274	0.0147	0.0071	0.0000	0.0000	0.0000
7	0.0526	0.0324	0.0182	0.0093	0.0000	0.0000
8	0.0835	0.0587	0.0377	0.0221	0.0118	0.0000
9	0.1127	0.0893	0.0648	0.0430	0.0262	0.0146
10	0.1319	0.1165	0.0947	0.0707	0.0485	0.0305
11	0.1360	0.1327	0.1197	0.0996	0.0764	0.0539
12	0.1251	0.1338	0.1329	0.1223	0.1040	0.0817
13	0.1038	0.1208	0.1312	0.1325	0.1242	0.1079
14	0.0783	0.0987	0.1164	0.1282	0.1315	0.1256
15	0.0541	0.0735	0.0936	0.1118	0.1248	0.1301
16	0.0344	0.0502	0.0688	0.0886	0.1071	0.1212
17	0.0202	0.0316	0.0464	0.0643	0.0837	0.1024
18	0.0109	0.0184	0.0289	0.0429	0.0600	0.0790
19	0.0053	0.0098	0.0166	0.0265	0.0397	0.0560
20	0.0022	0.0047	0.0088	0.0151	0.0242	0.0365
21	0.0008	0.0019	0.0041	0.0078	0.0136	0.0221
22	0.0002	0.0006	0.0017	0.0036	0.0070	0.0123
23	0.0000	0.0001	0.0005	0.0014	0.0032	0.0062
24	0.0000	0.0000	0.0001	0.0004	0.0012	0.0028
25	0.0000	0.0000	0.0000	0.0001	0.0004	0.0011
26	0.0000	0.0000	0.0000	0.0000	0.0001	0.0003
27	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001
28	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 6. Conditional Probability of t Reservations on Departure Given S Reservations Booked on the Review Day

t	S = 4	S = 5	S = 6	S = 7	S = 8	S = 9
4	0.0012	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0044	0.0020	0.0000	0.0000	0.0000	0.0000
6	0.0117	0.0063	0.0030	0.0000	0.0000	0.0000
7	0.0249	0.0152	0.0085	0.0042	0.0000	0.0000
8	0.0445	0.0303	0.0192	0.0112	0.0058	0.0000
9	0.0687	0.0512	0.0360	0.0236	0.0142	0.0076
10	0.0931	0.0755	0.0579	0.0419	0.0284	0.0176
11	0.1123	0.0985	0.0819	0.0645	0.0480	0.0334
12	0.1217	0.1149	0.1030	0.0876	0.0707	0.0540
13	0.1198	0.1212	0.1166	0.1067	0.0928	0.0767
14	0.1078	0.1164	0.1198	0.1174	0.1096	0.0973
15	0.0894	0.1027	0.1126	0.1177	0.1174	0.1117
16	0.0686	0.0836	0.0973	0.1083	0.1150	0.1167
17	0.0489	0.0631	0.0779	0.0919	0.1037	0.1118
18	0.0325	0.0443	0.0578	0.0722	0.0863	0.0987
19	0.0201	0.0290	0.0400	0.0528	0.0667	0.0808
20	0.0116	0.0177	0.0258	0.0360	0.0481	0.0615
21	0.0063	0.0101	0.0156	0.0230	0.0324	0.0437
22	0.0031	0.0054	0.0088	0.0137	0.0204	0.0290
23	0.0014	0.0027	0.0046	0.0076	0.0120	0.0180
24	0.0006	0.0012	0.0023	0.0040	0.0066	0.0105
25	0.0002	0.0005	0.0010	0.0019	0.0034	0.0067
26	0.0001	0.0002	0.0004	0.0008	0.0016	0.0029
27	0.0000	0.0001	0.0002	0.0003	0.0007	0.0014
28	0.0000	0.0000	0.0001	0.0001	0.0003	0.0006
29	0.0000	0.0000	0.0000	0.0000	0.0001	0.0002
30	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001

Table 7. Conditional Profit from Different Values of t on Departure if a Decision Has Been Made to Have Two Sections Available

t	Conditional Profit	t	Conditional Profit
4	- $405.	18	$245.
5	- $360.	19	$290.
6	- $315.	20	$335.
7	- $270.	21	$380.
8	- $225.	22	$425.
9	- $180.	23	$470.
10	- $135.	24	$515.
11	- $ 90.	25	$560.
12	- $ 45.	26	$605.
13	$ 0.0	27	$650.
14	$ 45.	28	$695.
15	$ 90.	29	$740.
16	$155.	30	$785.
17	$200.		

Table 8. Conditional Profit from Different Values of t on Departure if a Decision Has Been Made Not to Use Two Sections

t	Conditional Profit	t	Conditional Profit
4	- $195.	18	- $150.
5	- $150.	19	- $300.
6	- $105.	20	- $450.
7	- $ 60.	21	- $600.
8	- $ 15.	22	- $750.
9	+ $ 30.	23	- $900.
10	+ $ 75.	24	-$1050.
11	+ $120.	25	-$1200.
12	+ $165.	26	-$1350.
13	+ $210.	27	-$1500.
14	+ $255.	28	-$1650.
15	+ $300.	29	-$1800.
16	+ $150.	30	-$1950.
17	$ 0.0		

Table 9. Expected Profit from Making Either Decision When S Reservations Are Booked on the Review Day

S	Expected Profit from Decision		Decision
	Add a Second Section	Do Not Add a Second Section	
4	- $ 7.17	+ $103.58	Not to Add
5	+ $ 26.67	+ $ 96.29	Not to Add
6	+ $ 60.74	+ $ 77.50	Not to Add
7	+ $ 94.81	+ $ 47.50	Add
8	+ $129.10	+ $ 4.77	Add
9	+ $163.21	- $ 49.92	Add

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The conclusions that can be drawn from this study are as follows:

1. The conditional probability of t passengers on departure given S reservations booked on the review day must be considered in addition to the economic factors in the decision.

2. For any S, one of two courses of action will be chosen: (a) a second section will be added or (b) a second section will not be added, the choice depending upon the criterion of best decision.

3. The model developed in this study can provide an accurate guide for making the decision to add or not to add a second section when the required information is available and the conditions of the model are satisfied.

Limitations

The principal limitations of the study are expressed in the assumptions made to aid in the formulation of the mathematical model and the expected profit expressions. These assumptions were as follows:

1. Sufficient aircraft and flight personnel are available when they are needed and could be used for other flights when the second section is cancelled at departure time.

2. Additions to the number of reservations booked can be described by the Poisson distribution with an estimate of the parameter available.

3. Cancellations of the reservations can be described by the binomial distribution with an estimate of the parameter available.

4. A steady state condition exists for the probability of change in the number of reservations.

5. The parameters in the model do not change with different values for S_1 and S_2 within the values of interest.

6. If sufficient space is available, a passenger will not change accommodation status at departure.

7. The number of first-class passengers, t_1 or t_1', and the number of tourist-class passengers, t_2 or t_2', on departure are independent random variables.

Recommendations

In view of the limitations and conclusions of this study, the following recommendations are made:

1. That the revenues for a flight be studied to determine a more accurate value for considering the variations that exist in fares.

2. That a study be made to determine the magnitude of error involved in the assumption that t_1' and t_2' are independent events when a decision is made not to add a second section.

3. That a table or other aid be prepared to help reduce the computations involved in the solution of problems using the proposed decision rule.

4. That a procedure be developed for providing current estimates of the parameters of the model as they change.

5. That further study be undertaken to explore the application of this decision rule when applied to more than one flight in order that the allocation of available aircraft may be made as far in advance of departure as possible.

General Usefulness of the Results

The mathematical model for the reservation process and the decision rule for scheduling extra sections developed in this study should have many applications in situations where airlines or other forms of transportation have available aircraft or other vehicles and wish to take advantage of a source of revenue easily lost. The results of this study, however, are not offered as a panacea for the scheduling problem. In itself, the decision rule developed in this study is a suboptimization of the total scheduling problem. It is offered as a tool to assist the decision maker and to provide a method for evaluating the consequences of his decision on second sections.

An important feature of the results of this study is that by its nature, the decision rule developed is independent of the assumptions listed at the beginning of this thesis except for the necessity of estimating the values for the parameters of the model and the requirement for independence of the event, t_1' and t_2', occurring. If, for example, someone applying the decision rule to his particular problem found that the Poisson distribution did not accurately describe his process for new reservations, then a more suitable probability distribution could be substituted in the model. This substitution would in no way detract from the basic concepts incorporated in these results. This same conclusion would also apply to other changes. The substitution of other assumptions would only serve to refine the model to better describe a particular situation and improve on its usefulness.

The applications of this decision rule in the transportation industry are limited only to the extent that those individuals attempting to apply them are unable to provide the data necessary to evaluate the parameters and to satisfy the conditions of the model or make reasonable substitutions in the assumptions.

APPENDIX

ALGOL ROUTINE FOR PROBLEM

BAC-220 STANDARD VERSION 2/1/62

```
COMMENT DECISION RULE FOR AIRLINE W.A.REED I.E.DEPT.                $
INTEGER D,E,F,G,H,I,J,K,L,M,N,RA,P,W,X,Y,Z,U,C,S,A                  $
FLOATING OTHERWISE                                                  $
ARRAY(PP(20,10),MP(20),AP(30,10),R(23),NP(10),BP(40,10),
PPP(23,10,10),TT(50))                                               $
INPUT DATA1(D,F,H,J,L,N,U)                                          $
INPUT DATA2(FOR C=(1,1,D)$(MP(C)))                                  $
INPUT DATA3(FOR E=(4,1,F)$FOR G=(1,1,H)$(AP(E,G)))                  $
INPUT DATA4(FOR I=(1,1,J)$(NP(I)))                                  $
INPUT DATA5(FOR K=(4,1,L)$FOR M=(1,1,N)$(BP(K,M)))                  $
OUTPUT RESULT1(S)                                                   $
FORMAT COPY1(B6,*S=*,I3,B2,*.*,W4)                                  $
OUTPUT RESULT2(RA+(S-4),R(RA))                                      $
FORMAT COPY2(B10,I2,B11,X14.8,W0)                                   $
FORMAT HEAD2(B2,*NO. OF RESERVATIONS*,B8,*R(RA)*,W4)                $
OUTPUT RESULT3(Z+(S-4),TT(Z))                                       $
FORMAT COPY3(B10,I2,B11,X14.8,W0)                                   $
FORMAT HEAD3(B2,*NO. OF RESERVATIONS*,B8,*TT(Z)*,W4)                $
      READ($$DATA1)                                                 $
      READ($$DATA2)                                                 $
      READ($$DATA3)                                                 $
      READ($$DATA4)                                                 $
      READ($$DATA5)                                                 $
      FOR S=(4,1,4+U)                                               $
```

```
BEGIN  FOR C=(1,1,D) $ FOR P=(1,1,H) $ PP(C,P)=0.0                    $
       FOR C=(1,1,D) $ FOR P=(1,1,H) $ PP(C,P)=MP(C).AP(C+S-1,P)      $
       FOR RA=(4,1,23)                                                $
       R(RA)=0.0                                                      $
       FOR Y=(1,1,20)                                                 $
BEGIN  A=S-3  $  RA=(S+Y-A)                                           $
       IF Y LEQ 11  $  GO BAT  $  GO PAT                              $
BAT..  FOR C=(Y,1,Y+9)                                                $
       R(RA)=R(RA)+PP(C,C-Y+1)  $  GO TAT                             $
PAT..  FOR C-(Y,1,20)                                                 $
       R(RA)=R(RA)+PP(C,C-Y+1)                                        $
TAT..  END                                                            $
       FOR RA=(4,1,23)                                                $
       FOR I=(1,1,J)                                                  $
       FOR M=(1,1,N)                                                  $
       PPP(RA,I,M)=0.0                                                $
       FOR RA=(4,1,23)                                                $
       FOR I=(1,1,J)                                                  $
       FOR M=(1,1,N)                                                  $
       PPP(RA,I,M)=R(RA).NP(I).BP(RA+A-1+I-1,M)                       $
       FOR Z=(4,1,32)                                                 $
       TT(Z)=0.0                                                      $
       FOR Z=(4,1,32) $  BEGIN   IF Z EQL 4 $ GO TIP                  $
FOR RA=(4,1,Z-1) $ BEGIN IF RA GTR 23 $ GO XIXI                       $
FOR M=(1,1,10) $  BEGIN I=M+(Z-RA)                                    $
```

```
      IF  I  GTR 10  $  GO XIX                                     $
      TT(Z)=TT(Z)+PPP(RA,I,M)                                      $
XIX.. END $  XIXI..  END                                           $
TIP.. FOR RA=(Z,1,Z+9)                                             $
BEGIN IF RA GTR 23  $  GO PIT $ FOR I=(1,1,10)                     $
BEGIN M=I+(RA-Z) $ IF M GTR 10 $ GO XIM                            $
      TT(Z)=TT(Z)+PPP(RA,I,M)                                      $
XIM.. END $  PIT..  END  END                                       $
WRITE($$RESULT1,COPY1) $ WRITE($$HEAD2) $  FOR RA=(4,1,23)         $
      WRITE($$RESULT2,COPY2)                                       $
      WRITE($$HEAD3)                                               $
      FOR Z=(4,1,32) $  WRITE($$RESULT3,COPY3)  END                $
                      STOP                                         $
                      FINISH                                       $
      COMPILED PROGRAM ENDS AT 0933
      PROGRAM VARIABLES BEGIN AT 1218
```

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